A STUDY OF THE TECHNIQUE OF ELECTROCHEMILUMINESCENCE IN DETERMINING POINTS OF SEPARATION AND TRANSITION

by

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B.S., United States Naval Academy
(1956)

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SUBMITTED TO THE DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE MASTER OF SCIENCE DEGREE IN NAVAL ARCHITECTURE AND MARINE ENGINEERING AND THE PROFESSIONAL DEGREE, NAVAL ENGINEER

at the

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Department of Naval Architecture and Marine Engineering, May 21, 1965

Certified by . . . .

Thesis Supervisor

Accepted by . . . .

Chairman, Departmental Committee on Graduate Students

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by

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and

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Submitted to the Department of Naval Architecture and Marine Engineering on May 21, 1965, in partial fulfillment of the requirements for the Master of Science Degree in Naval Architecture and Marine Engineering and the Professional Degree, Naval Engineer.

ABSTRACT

Experiments were performed to study the technique of electrochemiluminescence for observing separation and transition in liquid flow over a body. The apparatus used was a gravity blowdown tunnel with two cylindrical test sections each 36 inches long and 2 inches and 4 inches in diameter, respectively.

The separation experiments were conducted on flat plates and cylinders. On the cylinders (0.5- and 1.5-inch diameter) a well-defined dark line was observed at approximately 89° from the leading edge in the Reynolds number range from 5.7×10^3 to 1.3×10^5. The lines were lines of separation as verified by photographs taken of smoke flow over one of the cylinders in a wind tunnel and also by comparing the position of the lines with experimental observations reported by Schlichting. A 5-inch long flat plate model was inclined at various angles of attack from 0.5° to 13.1° and a dark region was observed behind the leading edge in the Reynolds number range from 1.28×10^5 to 4.3×10^5 (based on a 5-inch length). Photographs taken of smoke flow over an inclined flat plate in a wind tunnel indicated that the dark region observed was the region of separation.

The transition studies were conducted by passing the chemiluminescent solution over a 30-inch long flat plate at zero angle of incidence with a maximum Reynolds number of 6.3×10^6. No phenomenon was observed that could be directly related to transition.

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This project was supported in part by the United States Navy and by a Shell Companies Foundation research grant to the Mechanical Engineering Department, Massachusetts Institute of Technology. This financial aid is gratefully acknowledged.

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a. Re = 17,100

b. Re = 45,300

c. Re = 106,000

d. Re = 130,000

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a. Re = 128,000

b. Re = 253,000

c. Re = 350,000

d. Re = 430,000

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a. Re = 128,000

b. Re = 253,000

c. Re = 350,000

d. Re = 430,000

FIGURE 14. SEPARATION ON FLAT PLATE AT VARIOUS ANGLES OF ATTACK. (REYNOLDS NUMBER BASED ON PLATE LENGTH OF 5 INCHES.) RE = 128,000 32

a. Angle of Attack = 0.5°

b. Angle of Attack = 2.5°

c. Angle of Attack = 3.2°

d. Angle of Attack = 13.1°

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b. Angle of Attack = 15°, Re = 2,400

c. Re = 735

I. INTRODUCTION

It has long been recognized that fluid flow visualization has played an important part in the development of the science of Fluid Mechanics.[1*] Electrochemiluminescence offers the possibility of a new flow visualization technique. It is novel in that events in the boundary layer are observed directly, and no extraneous object is introduced into the flow. Chemicals are added to the water, but these are dissolved, and the resulting fluid is a homogeneous solution.

Howland, et al.,[2] were the first to apply the technique of electrochemiluminescence to fluid flow visualization studies. In this technique a model coated with platinum is immersed in an alkaline solution of Luminol and Hydrogen Peroxide.[**] The platinum on the model serves as one electrode (the anode), and the other electrode (the cathode) consists of a sheet of aluminum. A continuous blue glow is observed on the surface of the platinum when the potential between the electrodes is approximately one volt D. C. and relative motion exists between the fluid and model. Then patterns are observed in the blue glow which apparently represent flow phenomena (e.g., separation). When the potential is increased to approximately five or six volts between electrodes, the glow patterns tend to detach from the surface of the platinum and trail in the wake of the model. This study treats only phenomena which are observed at the surface of the model.

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[*] Superscripts indicate references at the end of the thesis.

[**] A detailed description of the solution is given in Appendix A.

Howland, et al.,[2] working with a small apparatus (essentially a large rotating bowl containing the chemiluminescent solution) observed what they considered were regions of separation and transition on cylinders and flat plates. Schiller[3] designed and built an apparatus with a capacity for achieving higher Reynolds numbers than were possible using Howland's apparatus. Schiller, experimenting with flat plates, believed that he was observing separation at various angles of attack on the plates. Springer[4] showed that in laminar flow the light intensity is governed by mass transfer of the chemiluminescent substance to the surface of the platinum.

The above observations suggest that there is a correlation between the light intensity, observed glow patterns, and events occurring in the boundary layer. This study was undertaken as an attempt to correlate observed glow patterns with events occurring in the boundary layer with particular emphasis on separation and transition. Attempts will be made to observe what are considered to be separation and transition points in the blue glow on various models. The observed points will be compared to existing experimental data and theoretical results where possible to determine whether or not the technique of electrochemiluminescence can be used as a valid flow visualization technique for transition and separation.

II. PROCEDURE

A. Experimental Apparatus

The apparatus used in this study is that designed and built by Schiller.[3] It is a gravity blowdown system and is shown in Figure 1. The test section (shown in Figure 2) is a plexiglass tube approximately 36 inches long. The inside diameter of the tube is 4 inches. The model to be studied (e.g., flat plate or cylinder) is placed in the test section, and the test section is then secured in its proper position in the apparatus as shown in Figure 1. The settling tank is filled to the desired level (depending on the length of test run desired) with chemiluminescent solution (the details of which are given in Appendix A), and the anode to cathode potential is established by turning on the D. C. power supply. The power supply used in this study was a Harrison Laboratories, Inc., model 880A Regulated Power Supply with a range of from 0-100 volts and 0-1 amperes. When the control valve is opened, the chemiluminescent solution flows over the model being studied, and photographs are taken of the resulting glow patterns.

The apparatus was designed to approximate slug flow in the test section. During the initial phase of this study, it was necessary to determine whether or not slug flow does in fact exist in the test section. To do this the test section was instrumented with two pitot tubes as shown in Figure 2. The pitot tubes are made of 1/16-inch inside diameter stainless steel tubing. The outside diameter is 3/32 of an inch. The lengths of the tubes are 3-3/4 inches, and their position across the test section is adjustable so that complete coverage

of over half of the test section profile is obtained. The tubes are bent to an angle of 90 degrees one inch from one end. The radius of the bend is 3/16 of an inch. The pitot tubes measure the dynamic pressure only. The static pressure is measured through a static pressure tap which is a 3/32-inch hole in the side of the test section adjacent to the pitot tube. The settling tank was filled to the top with approximately 1500 liters of water, and the velocity profile across the test section was measured as the water was released. The measured velocity profiles verified that nearly slug flow exists everywhere in the test section. Figures 3 and 4 show the velocity profiles obtained with the settling tank full (1500 liters) at the start of the run. The maximum velocity of the water in the test section was approximately 10.8 feet per second. Figures 5 and 6 show the velocity profiles obtained when the procedure given above is repeated with the settling tank half full (approximately 750 liters) of water at the start of the run. For this case the maximum velocity of the water in the test section was approximately 10.4 feet per second.

It was determined that the flow is somewhat time dependent, but in any given test run after time t = 10 seconds (where t is time after opening the control valve), the velocity vs. time characteristics are flat. They approximate steady state conditions since the velocity decreases .01 feet per second per second after time t = 10 seconds. Figures 7 and 8 show the velocity vs. time characteristics for runs with the settling tank full and half full, respectively, at the start of the run. The total run time with a full settling tank is 54 seconds, and the total

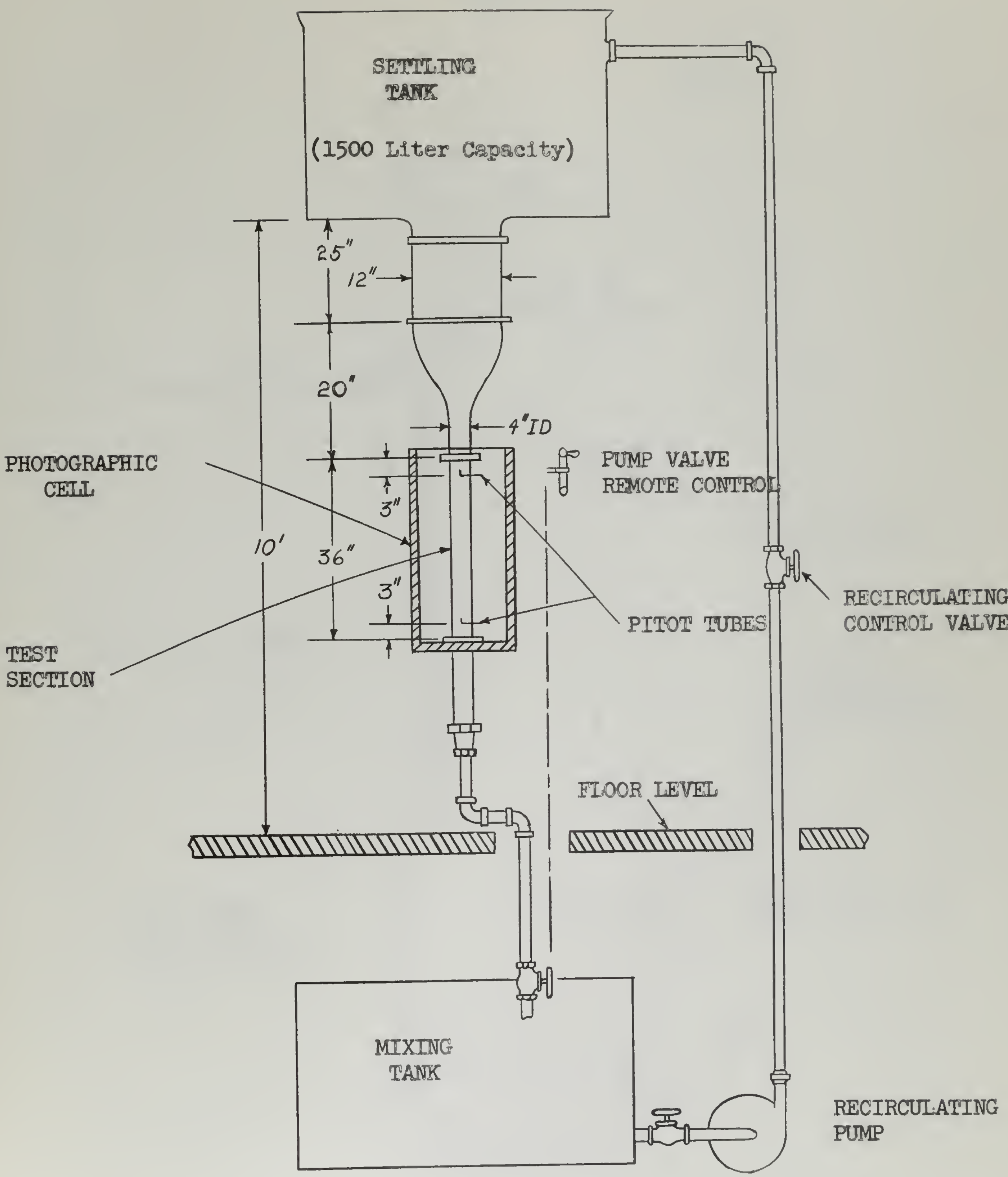


FIGURE 1 TEST APPARATUS

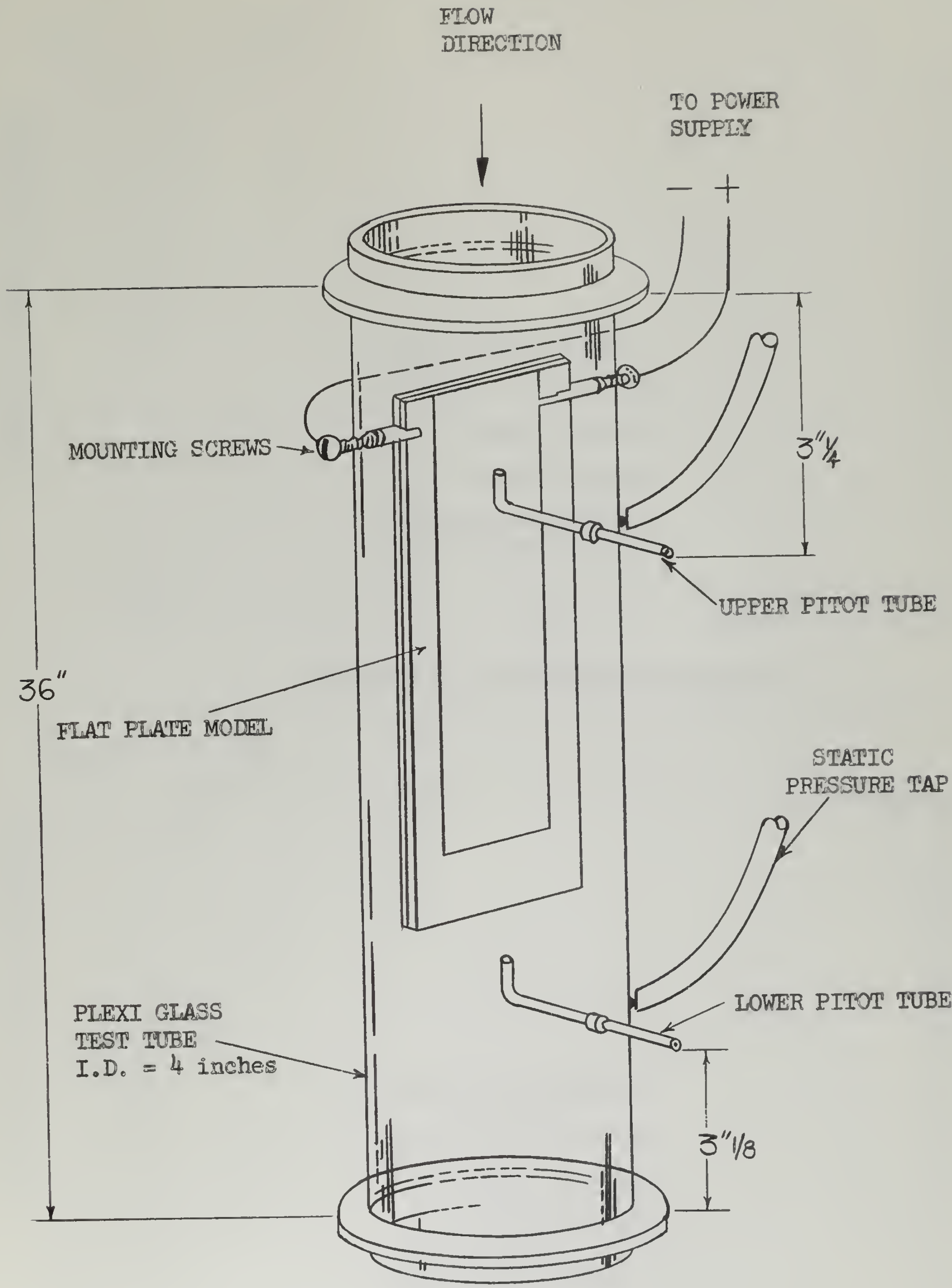
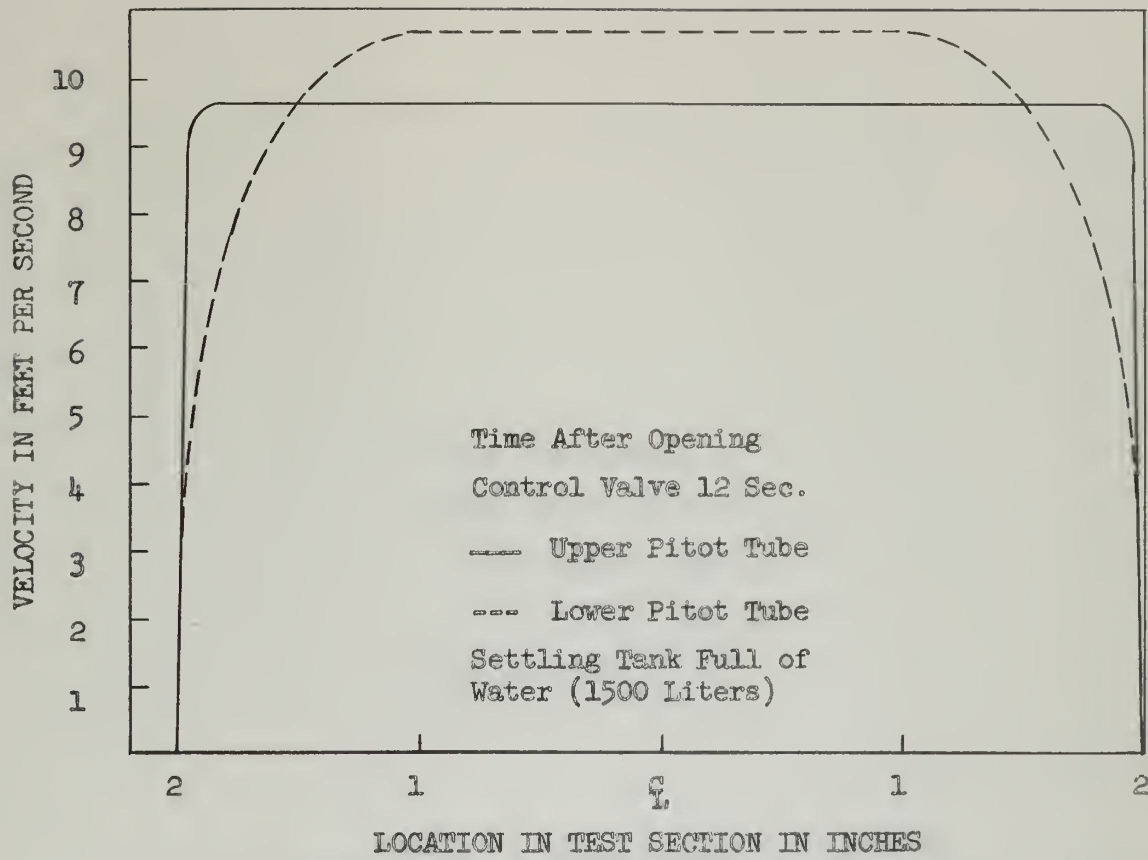


FIGURE 2 TEST SECTION



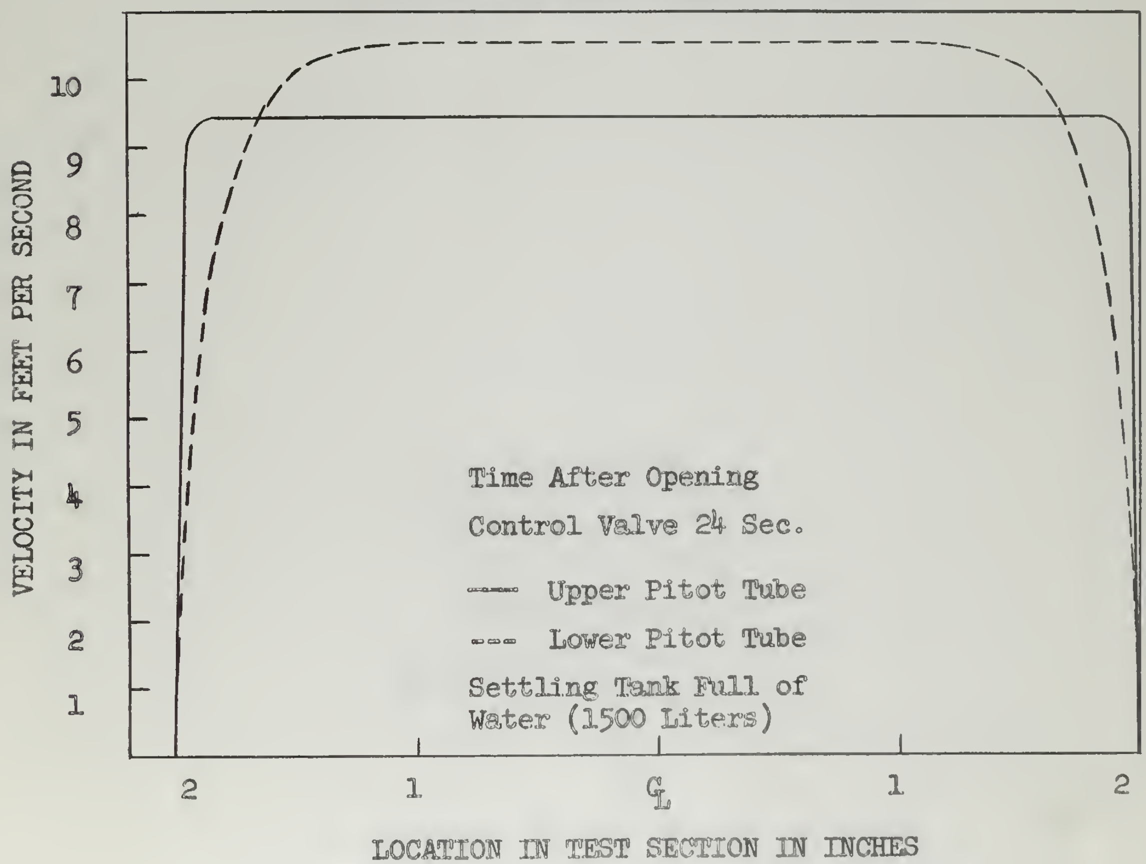
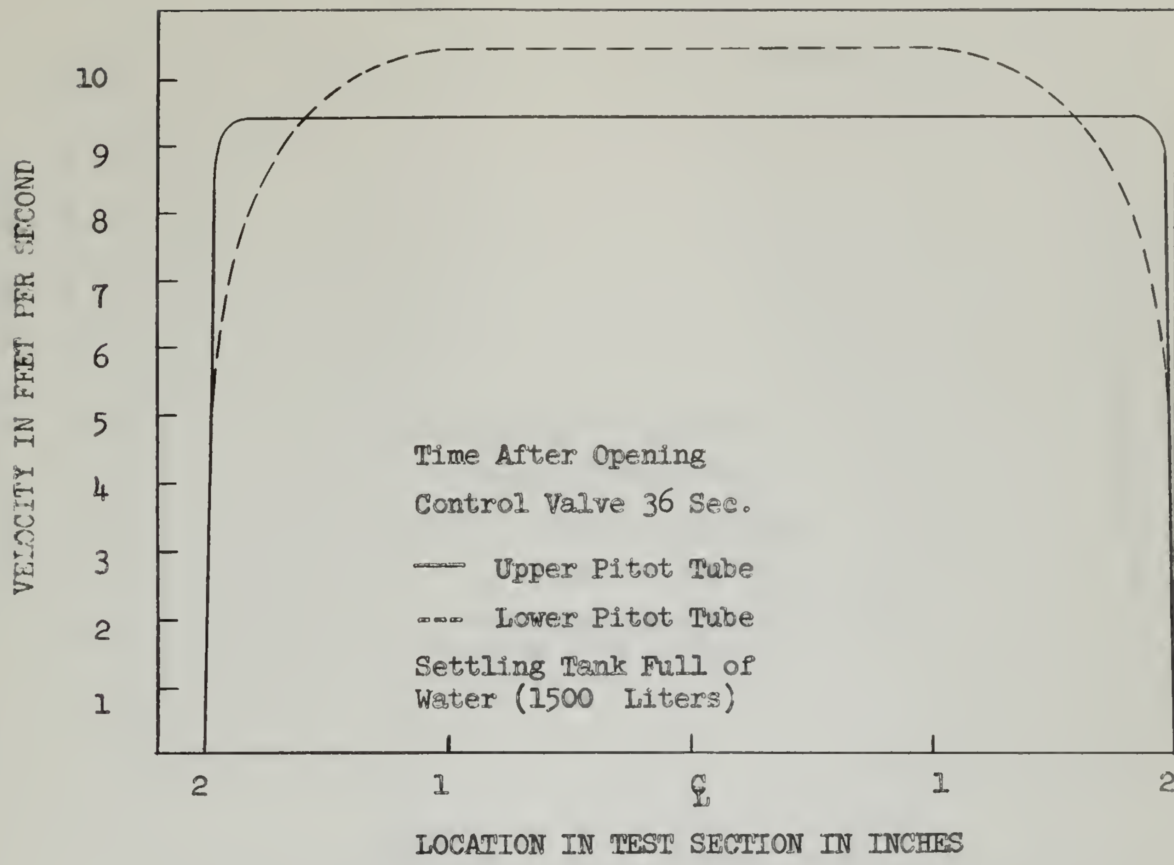


FIGURE 3 VELOCITY PROFILES IN TEST SECTION



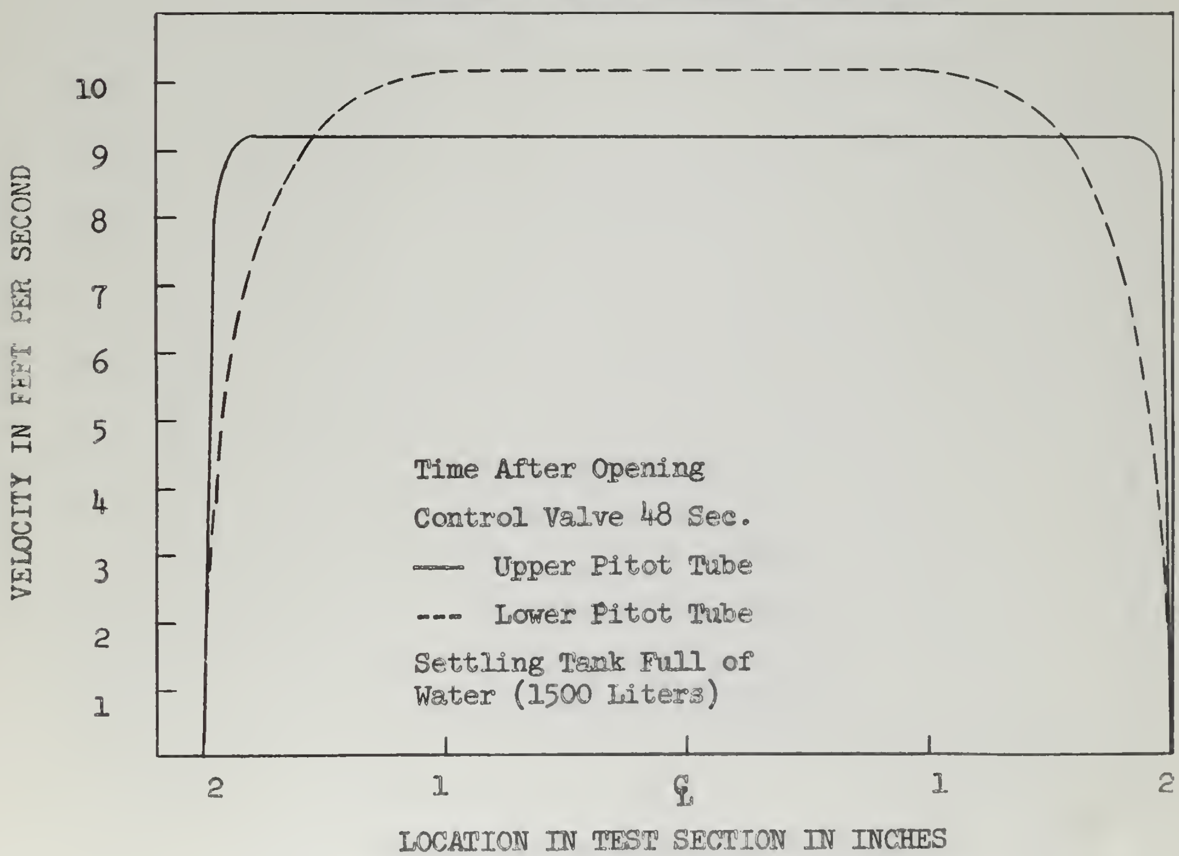
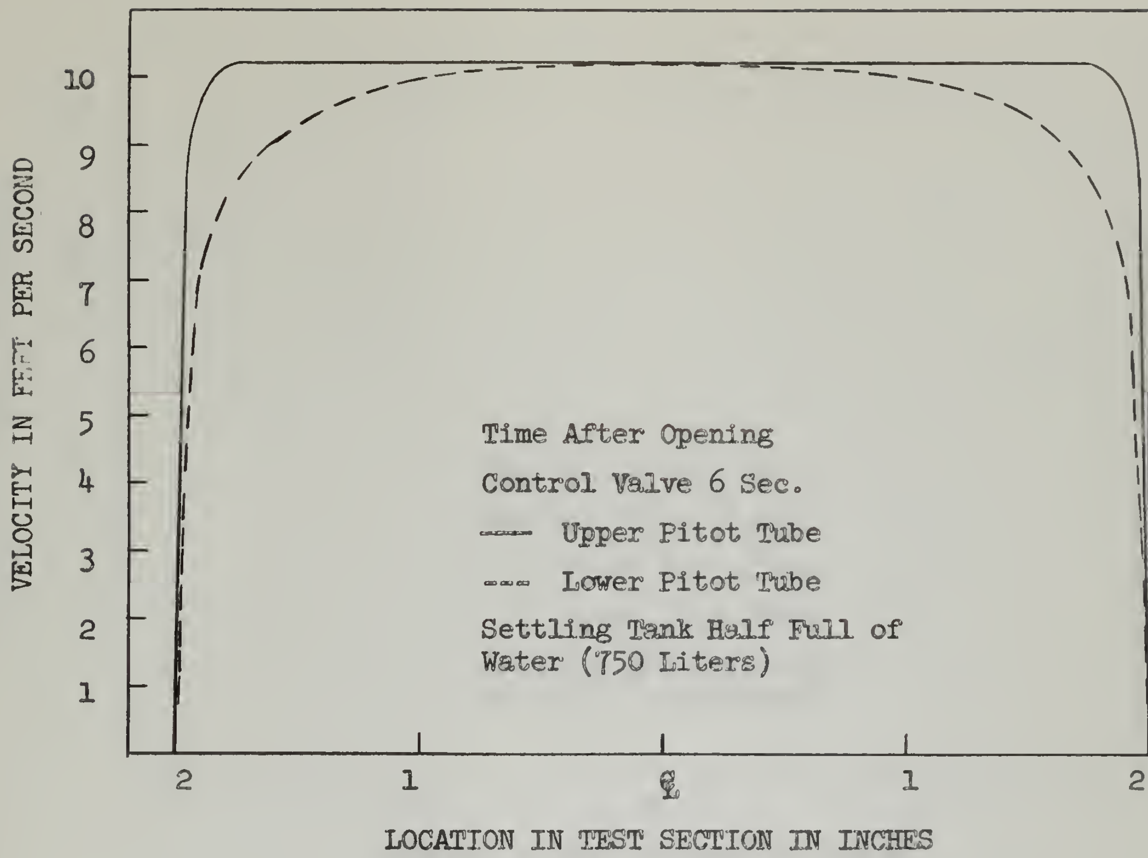


FIGURE 4 VELOCITY PROFILES IN TEST SECTION



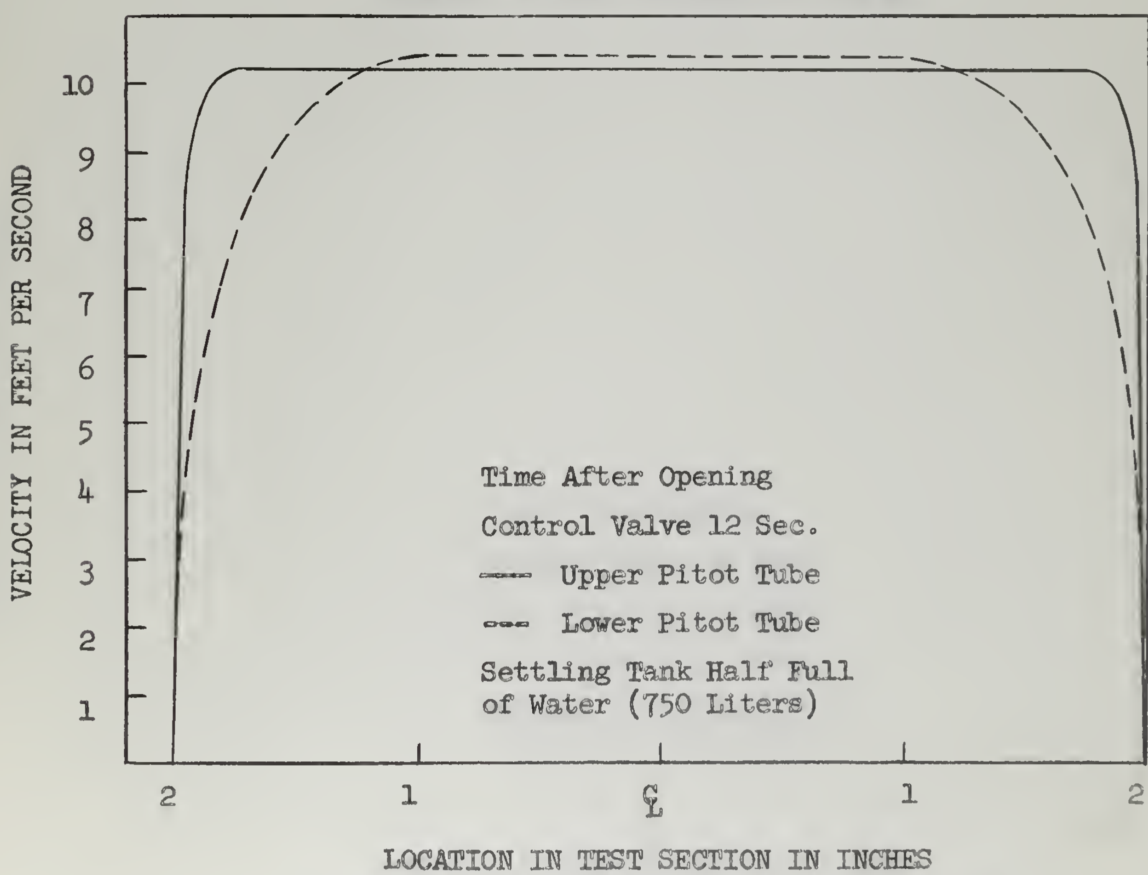
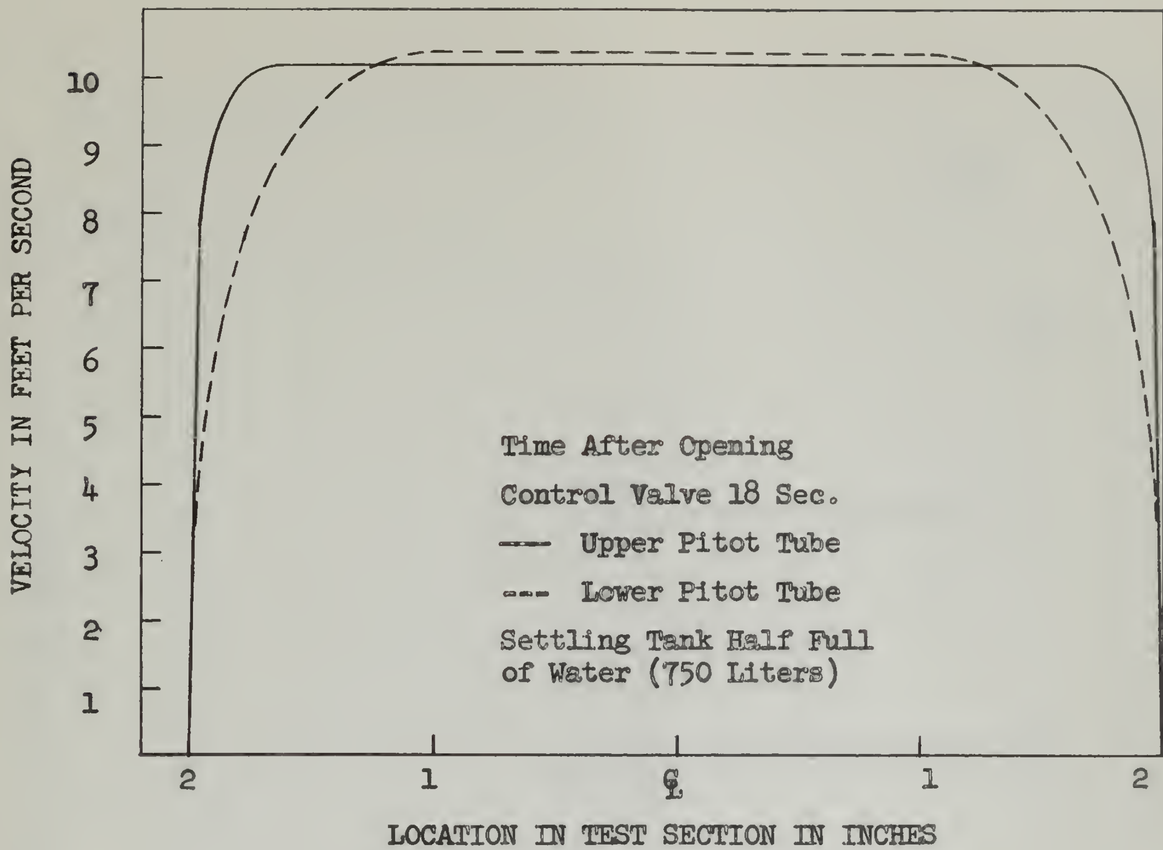


FIGURE 5 VELOCITY PROFILES IN TEST SECTION



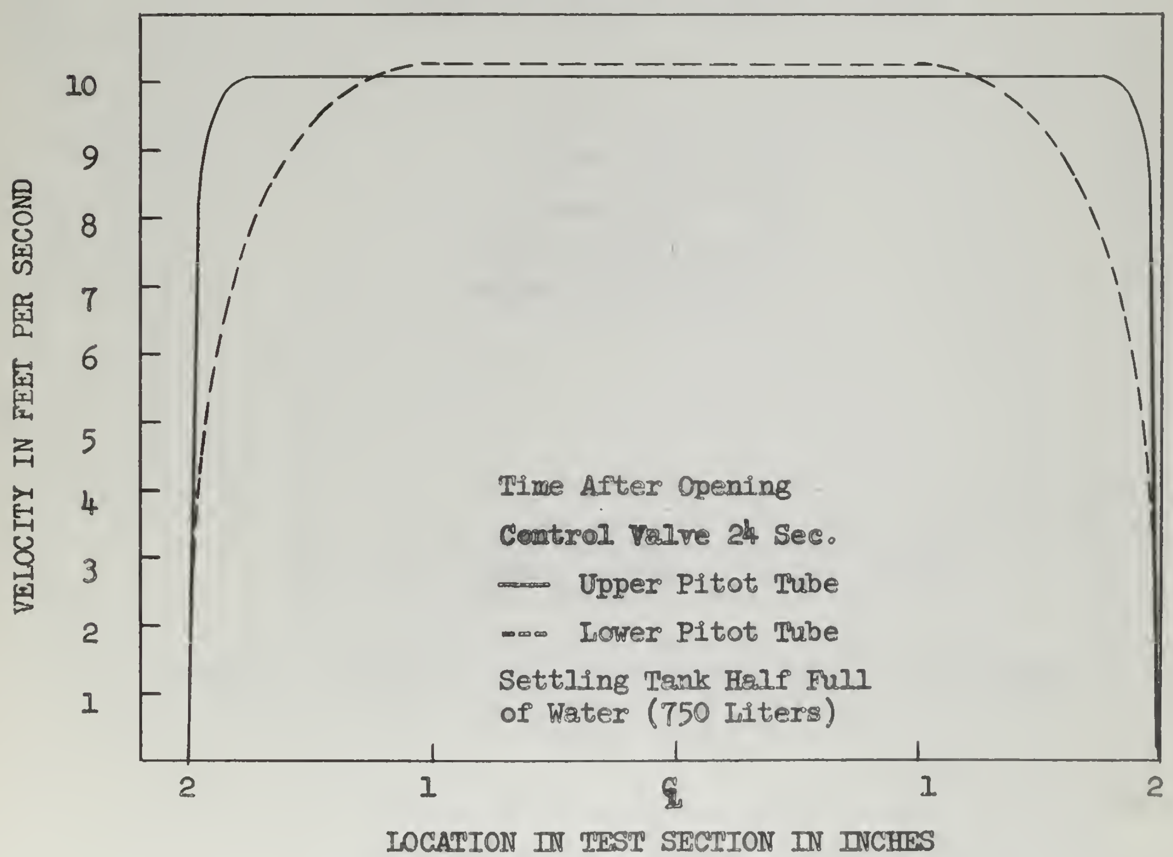


FIGURE 6 VELOCITY PROFILES IN TEST SECTION

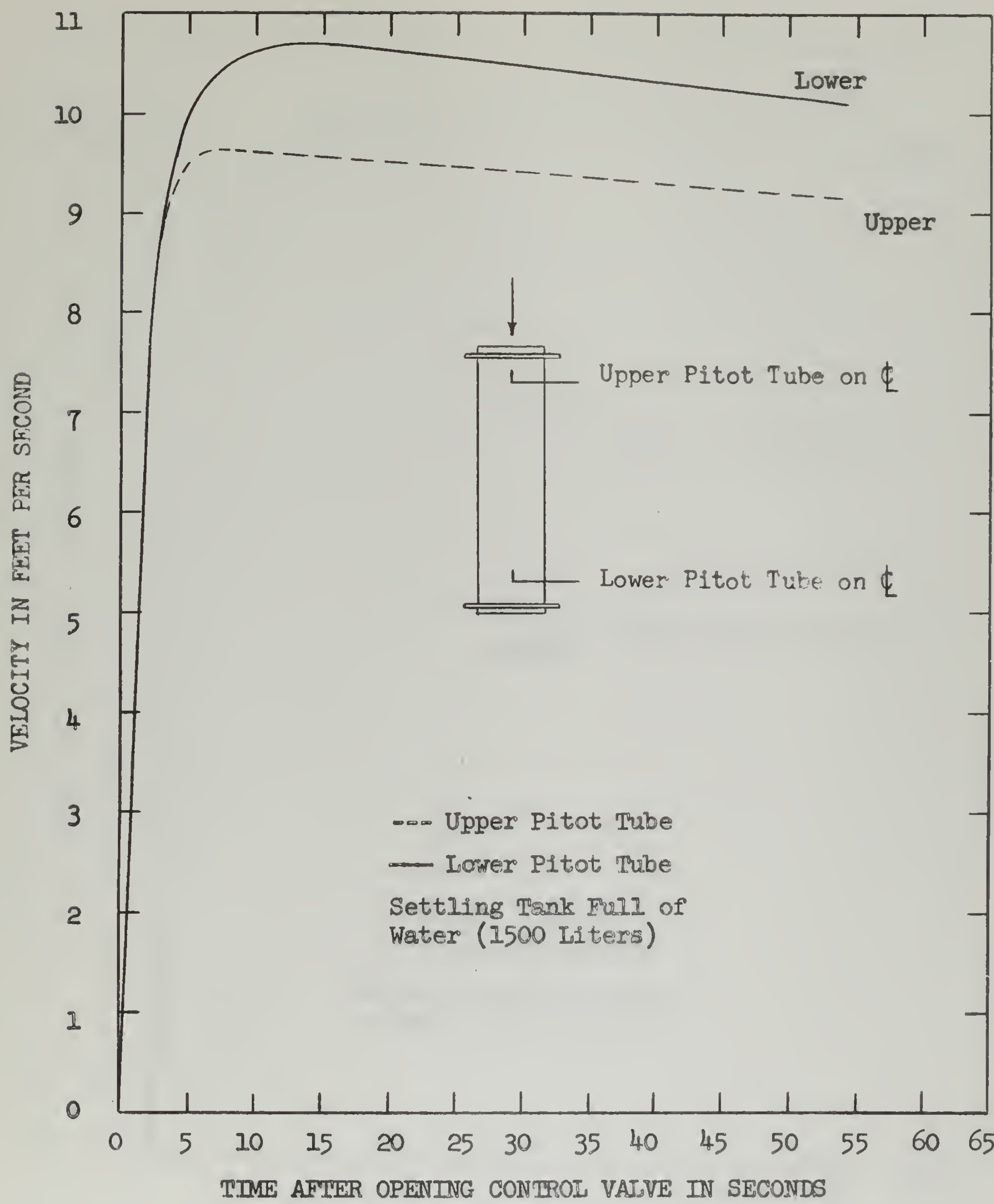


FIGURE 7 VELOCITY - TIME CHARACTERISTICS IN TEST SECTION

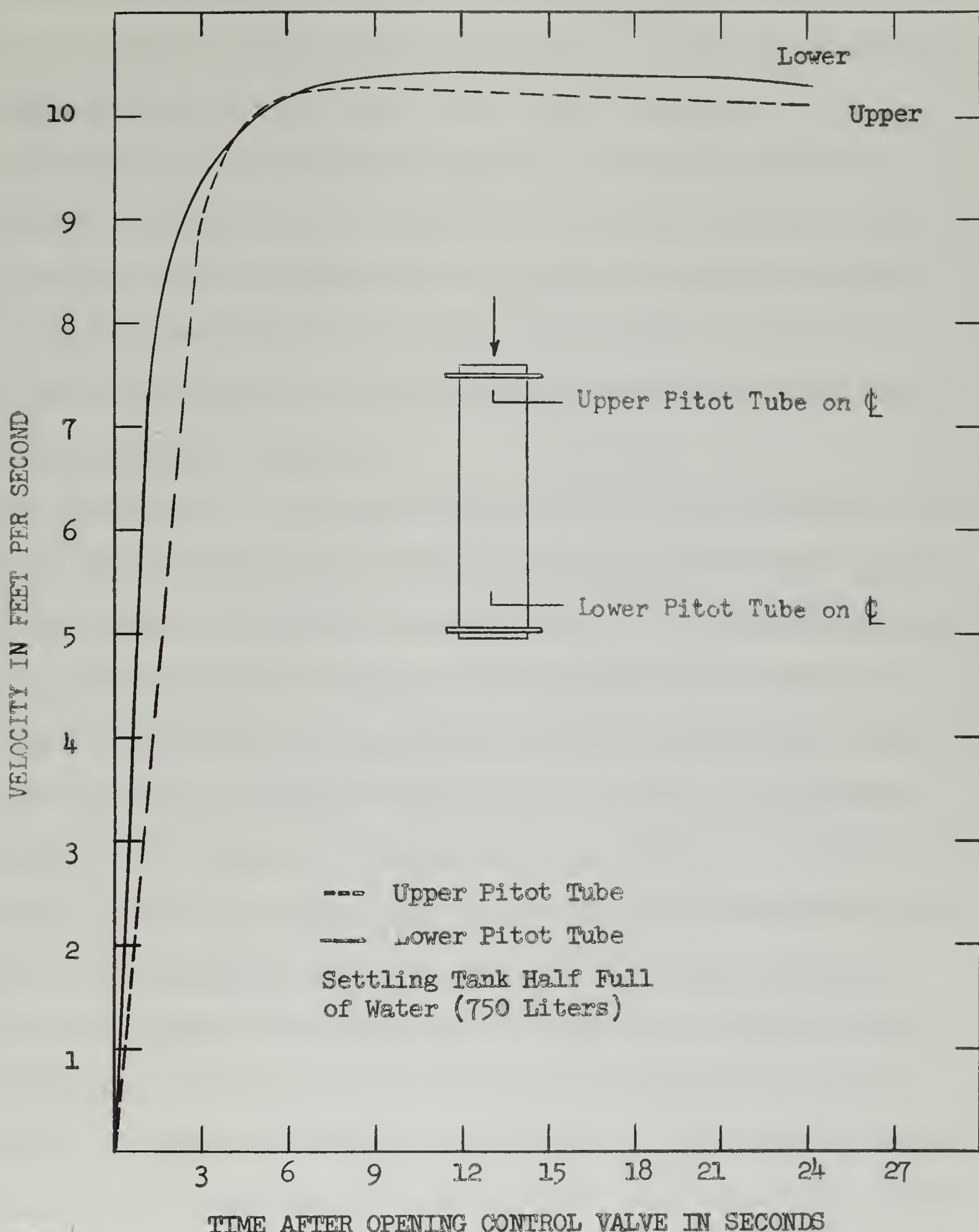


FIGURE 8 VELOCITY - TIME CHARACTERISTICS IN TEST SECTION

time with a half tank is 24 seconds. In the interests of economy and time, all actual test runs with chemiluminescent fluid were made with the settling tank one-half full at the start of the run.

The electrical system used is the same as that described by Schiller.[3] It consists of the model either platinum plated or coated with platinum foil as the anode and an aluminum plate as the cathode. The flat plate models used are plexiglass flat plates with platinum foil glued to one surface of the plate and an aluminum plate glued to the other surface. The glue used is a two-part epoxy glue. One cylindrical model used is a 1.5-inch diameter stainless steel cylinder. The cylinder face is electroplated with approximately 0.002 inch of platinum. The ends of the cylinder are not plated. The other cylinder model is a 0.5-inch diameter plexiglas rod with platinum foil glued to its surface. The electrical connections are made through the two screws in the test section (Figure 2) when flat platemodels are in the test section. In the case of cylinders the anode connection is made through one screw in the test section, and the cathode connection is made with a small wire attached to the aluminum plate and running out the back of the test section. The cylinder cathode is an aluminum plate which is suspended in the test section and not attached in any way to the cylinder. The anode and cathode are connected to the + and - terminals, respectively, of a D.C. variable power supply. The power supply serves as a switch as well as the means for varying the voltage. During the actual test runs the voltage was adjusted to maintain the maximum light intensity without generating detached glow patterns. It was determined that approximately 3 to 4 volts anode to cathode potential were required.

Late in the study it was determined that the glow on the anode could be obtained if the platinum were replaced with stainless steel. The glow obtained is as intense as that using platinum as the anode. In this case, however, the KOH content of the solution was 0.1 Normal instead of 0.01 Normal as recommended by Howland.[2] It is not certain what effect the added KOH had on the solution or the electrochemiluminescent process other than to make the solution more alkaline. No attempt was made to investigate the solution or anode material further.

B. Electrochemical Solution

The composition of the solution used is that specified by Schiller.[3] This solution differs from Howland's[2] in that only one third of the amount of potassium chloride is used. Schiller determined that this reduction of KCl makes no apparent change in the glow intensity. The solution is composed of luminol, hydrogen peroxide, potassium chloride, potassium hydroxide, and purified water. The details of the amounts and functions of each chemical appear in Appendix A.

It is necessary to state here that the water must be purified. This is done because an excessive concentration of iron or copper impurities can cause bulk glow in the vicinity of the test piece. Either distilled water or water purified by ion exchanger cartridges can be used. The ion exchangers (Barnstead Standard Type 0802) have been used because they are the most inexpensive and convenient way of obtaining the large amount of purified water required (1500 liters for a full tank).

In the early stages of the study, a small Howland[2] type apparatus was constructed for familiarization with the technique. In addition

the effect of replacing KCl with NaCl was investigated. The sodium chloride appeared to provide the same glow intensity as the potassium chloride. It was further determined that the amount of NaCl could be reduced to one third its original amount without any apparent loss of intensity. Finally it was determined that the same results could be obtained with technical grade NaCl substituted for the reagent grade. These factors will provide a financial savings for future investigators. In this study potassium chloride was used because of the large quantity on hand at the outset.

III. DISCUSSION OF RESULTS

A. General

In this study separation and transition were the flow phenomena of primary interest. It was expected that using circular cylinders and flat plates as models, visual evidence of separation and transition would be obtained. The glow pattern should appear as an even blue glow on the surface of the anode in laminar flow up to the point of separation. The glow should disappear along the separated region giving the appearance of a black line or region against the blue glow. Transition was expected to take the form of wavy regions in the glow or perhaps an increased light intensity beginning at the transition point.

B. Circular Cylinder

The circular cylinder was chosen as a model to be studied because of its simple geometry. The fact that some experimental data is available for the separation point on a cylinder in laminar flow also played a part in the decision to use a cylindrical model.

Test runs were conducted on two cylinders. One cylinder was 1.5 inches in diameter and 2 inches long; the other cylinder was 0.5 inch in diameter and 4 inches long. The tests were run to determine if a line of separation could be distinguished in the glow on the face of the cylinder. A line believed to be the line of separation was seen, and Figure 9 shows this line on the 0.5-inch and the 1.5-inch diameter cylinders. The line on the 1.5-inch diameter cylinder was seen to occur at approximately $\phi = 89^{\circ}$ where ϕ is the angle measured from the

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Schlichting[5] cites the work of Hiemenz who in his thesis to Goettingen University in 1911 reported measurements of the pressure distribution around a cylinder in steady flow. Using the measured pressure distribution as a basis, he calculated that ϕ at separation

should be 82 degrees. He observed ϕ at separation on a cylinder to be 81 degrees. Schlichting also mentions the work of Flachsbert who experimented with cylinders in 1932. He determined that if the Reynolds number (based on cylinder diameter) were below a certain critical value (3×10^5), then the pressure minimum on the cylinder occurs at approximately $\phi = 70^\circ$, and separation would almost certainly occur before $\phi = 90^\circ$. If the Reynolds number were above the critical value, then the pressure minimum would occur near $\phi = 90^\circ$, and separation would occur approximately as indicated by potential theory. In the present study the highest Reynolds number obtained over a cylinder was 1.3×10^5 which is below the critical value, and hence it can be expected that separation will occur before $\phi = 90^\circ$.

Using Thwaites'[12] method (see Appendix B) with the ideal flow velocity distribution around a cylinder, the value of ϕ at separation was calculated. The value obtained ($\phi = 102^\circ$) is somewhat greater than the observed $\phi = 89^\circ$. This can be attributed to using the potential steady flow velocity distribution around the cylinder instead of a measured velocity or pressure distribution.

As a final confirmation, photographs were taken of smoke flow over a 1.5-inch diameter cylinder in a wind tunnel. The Reynolds number was 735. Figure 15c shows the result of smoke flow over the cylinder and confirms that separation occurs with ϕ between 85 and 90 degrees. It is concluded that the line observed on the cylinders used in this study is in fact the laminar flow line of separation.

C. Flat Plate

The flat plate was chosen as a model for study primarily because it was expected that a separated region would be visible at or near the leading edge. It was also hoped that reattachment of the separated flow to the plate might be observed.

Test runs were conducted on a flat plate model 5 inches long and 2-1/4 inches wide. The Reynolds number (based on a plate length of 5 inches) was varied as was the angle of attack. The test runs were conducted to determine if a line or region of separation could be distinguished in the glow over the flat plate.

A region, believed to be the separated region, was observed from the leading edge to approximately 0.1 to 0.2 inch downstream from the leading edge. Figures 12, 13, and 14 show this region for various angles of attack and Reynolds numbers. These observations suggest that separation is taking place at the leading edge of the flat plate, as expected. Photographs were taken of smoke flow over a flat plate in a wind tunnel to attempt to confirm that separation does occur at the leading edge. Figure 15a and b shows this flow at angle of attack 10 degrees and 15 degrees, respectively, at a Reynolds number of 2.4×10^3 (based on a flat plate length of 5 inches). It is seen that separation does in fact occur at the leading edge of the flat plate.

Immediately following the dark separated region is a bright region across the width of the plate. This region is almost a line, and it is believed that reattachment of the flow to the plate occurs here. Bourque and Newman[6] experimented with small jets of fluid flowing over

inclined flat plates. They observed separation at the leading edge of the flat plates and reattachment downstream from the leading edge. The small jets (0.187 inch in diameter and smaller) flowing over inclined plates do not exactly correspond to the circumstances of the present study where a 4-inch diameter circular fluid channel is provided for flow over models. The general circumstances of incompressible, two-dimensional flow over flat plates is the same, however, and qualitative comparisons should be valid. Bourque and Newman[6] experimented with angles of attack of 30 degrees and greater. They showed that separation occurs at the leading edge of the flat plates which is in agreement with the observations of the present study. They showed that reattachment occurs further downstream from the leading edge as the angle of attack is increased. The experimental results of Bourque and Newman when extrapolated to the angles of attack used in the present study (0.5 degree to 13.1 degrees) indicate that reattachment occurs very near the leading edge of the plate. This is in agreement with the observations of the present study. Figure 14 shows the flow over a flat plate at various angles of attack at Reynolds number (based on flat plate length of 5 inches) of 1.28×10^5. It can be seen that at angles of attack of 0.5 degree, 2.5 degrees, and 3.2 degrees (Figures 14a, b, and c, respectively), reattachment occurs at approximately the same location near the leading edge of the flat plate, but as the angle of attack is increased to 13.1 degrees (Figure 14d) reattachment occurs further downstream from the leading edge. On the basis of this qualitative evidence, it is concluded that reattachment of the flow to a flat plate is observable using the technique of electrochemiluminescence.

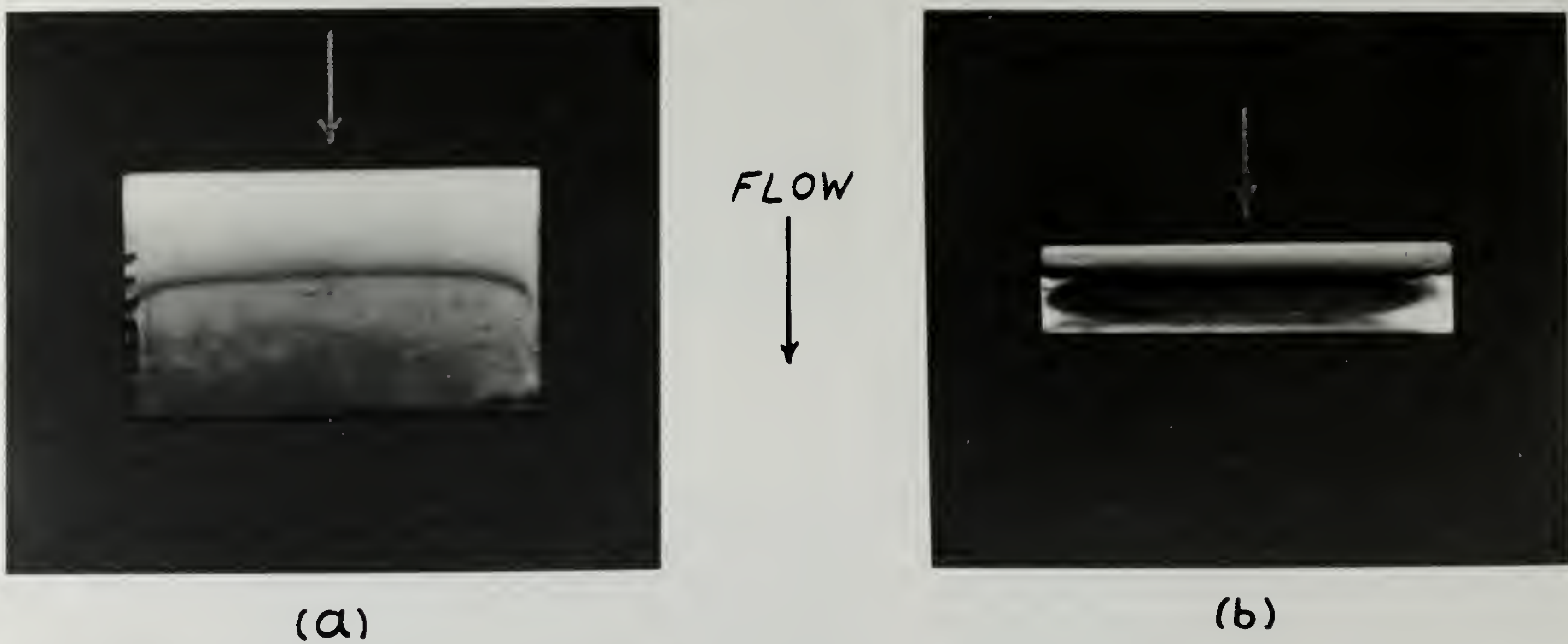


FIGURE 9. Line of separation on cylinders. Diameter (a) d = 1.5 inches, (b) d = 0.5 inches. Reynolds number (based on diameter) (a) Re = 17,100 , (b) Re = 5,700 .

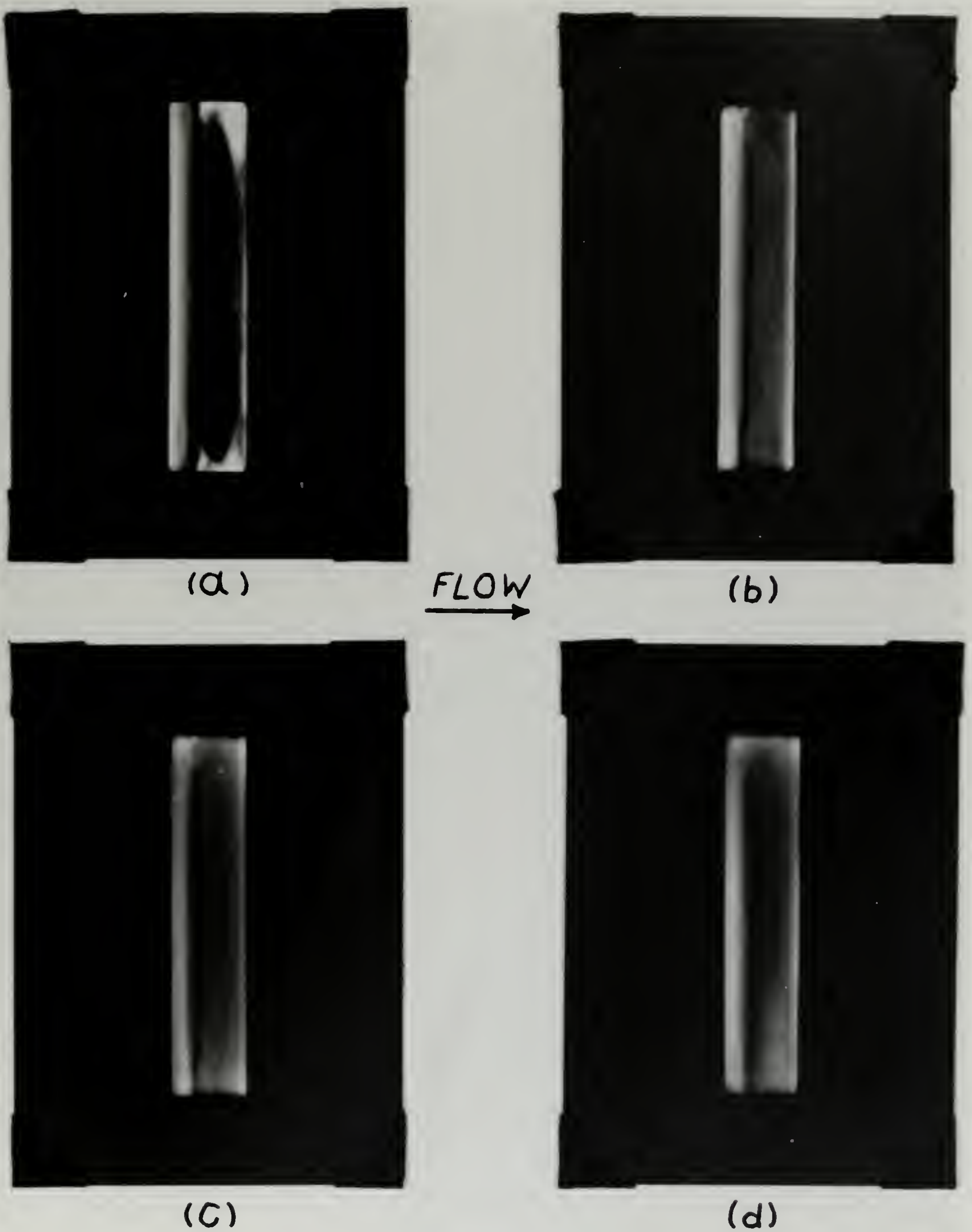


FIGURE 10. Line of separation on cylinders at various Reynolds numbers (based on cylinder diameter of 0.5 inches) (a) Re = 5,700 (b) Re = 15,100 (c) Re = 35,300 (d) Re = 43,300 .

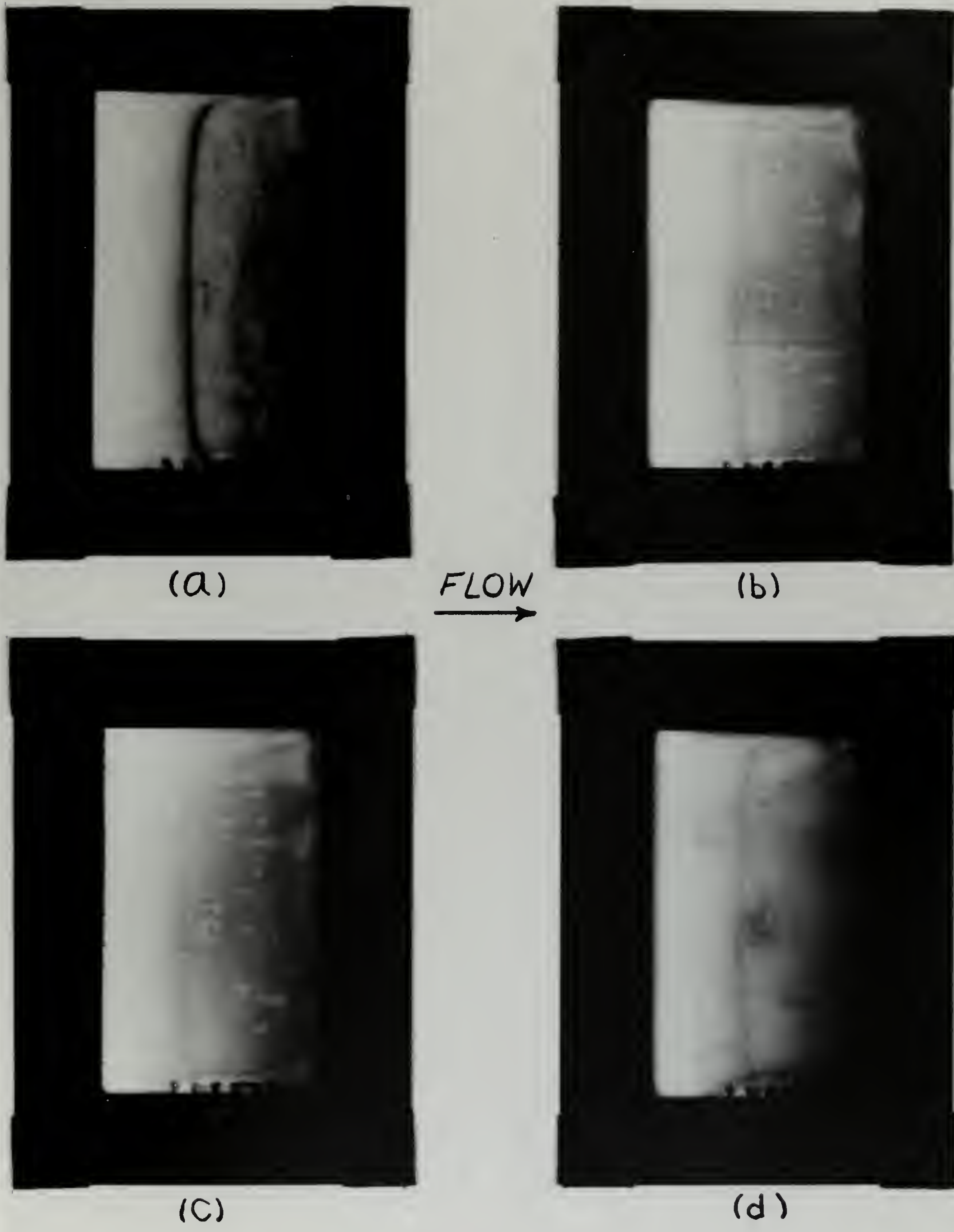


FIGURE 11. Line of separation on cylinders at various Reynolds numbers (based on cylinder diameter of 1.5 inches) (a) Re = 17,100 (b) Re = 45,300 (c) Re = 106,000 (d) Re = 130,000 .

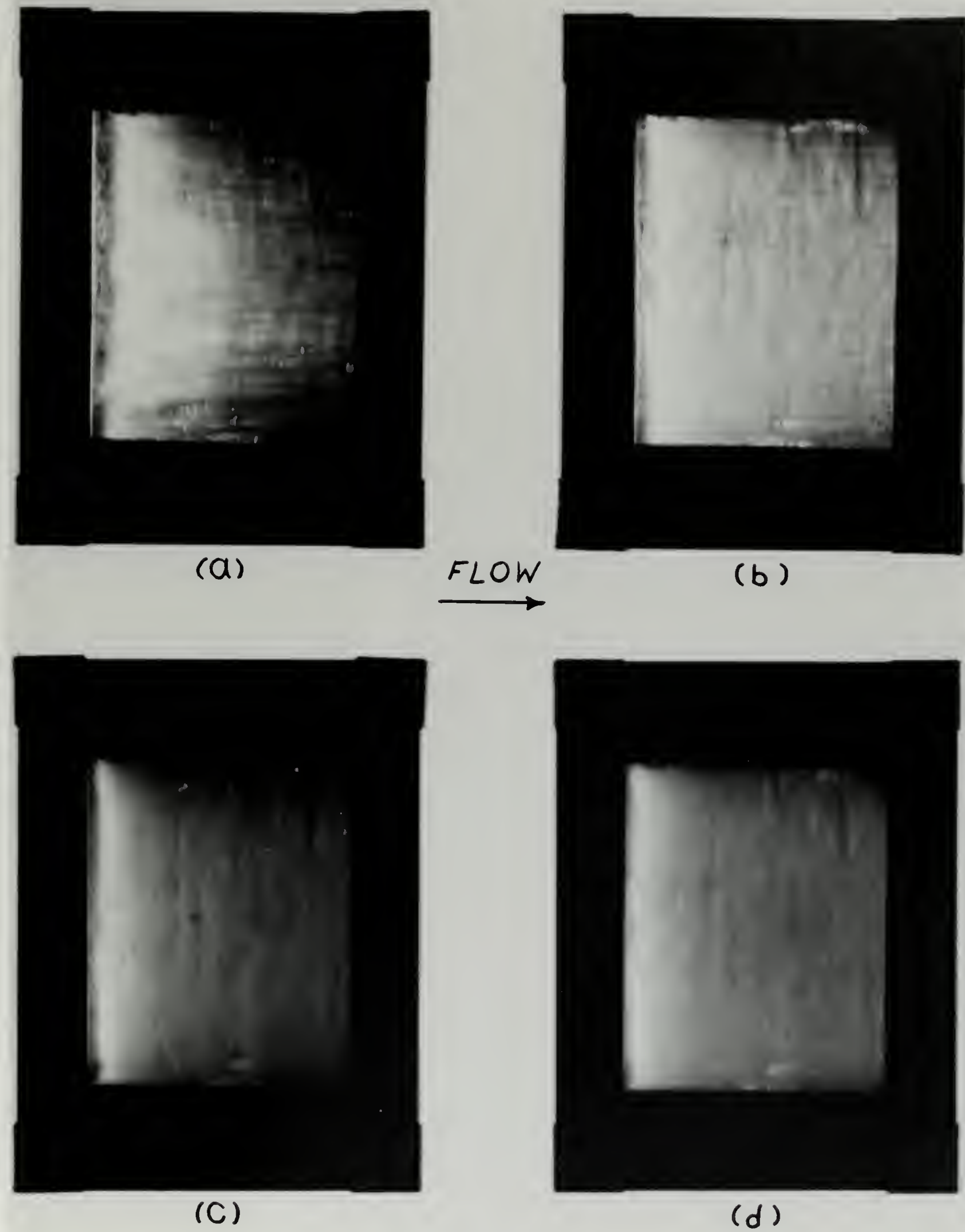


FIGURE 12. Separation on a flat plate at various Reynolds numbers. Angle of attack = 0.5° . Reynolds number (based on plate length of 5 inches) (a) Re = 128,000 (b) Re = 253,000 (c) Re = 350,000 (d) Re = 430,000 .

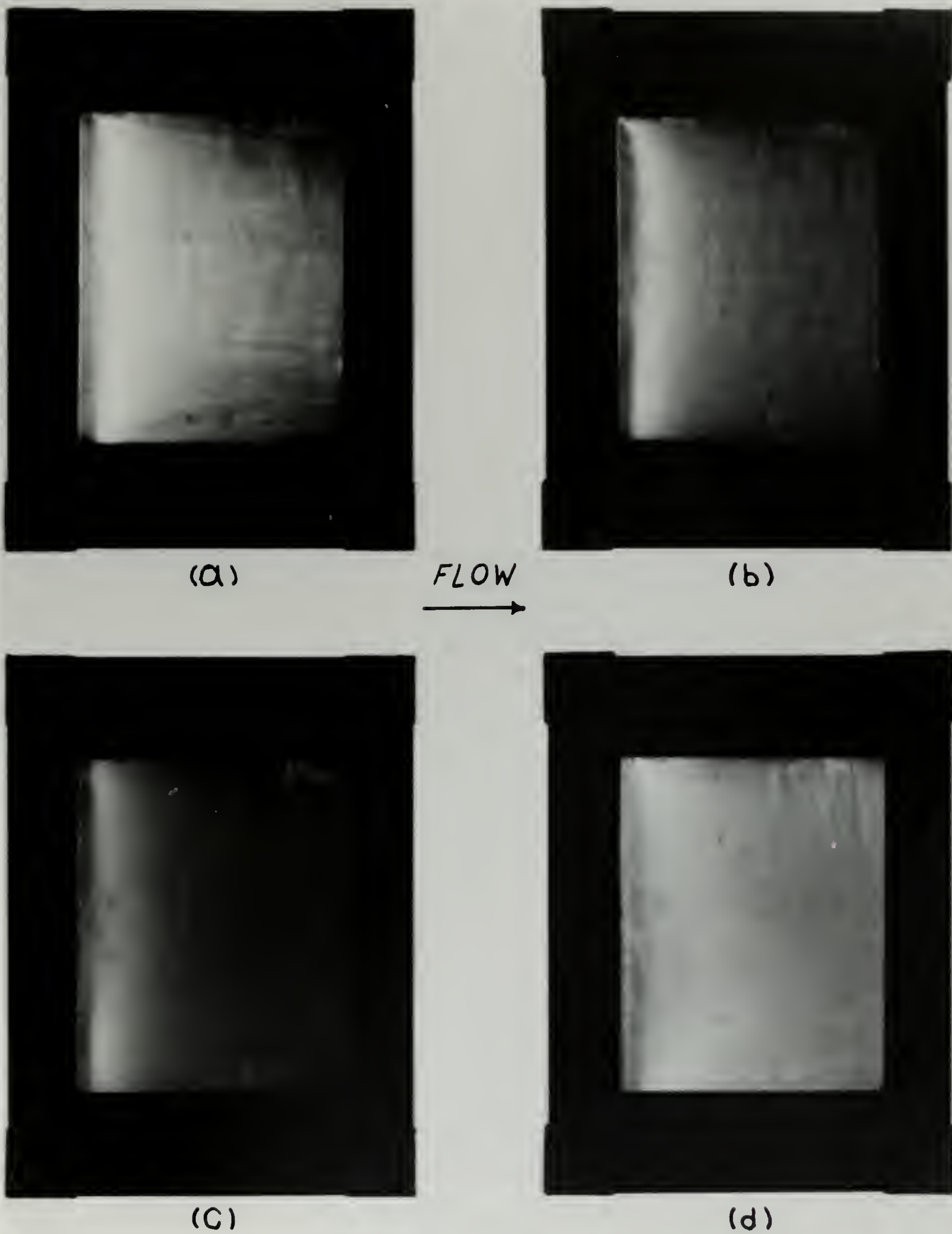


FIGURE 13. Separation on a flat plate at various Reynolds numbers. Angle of attack $= 2.5^{\circ}$. Reynolds number (based on plate length of 5 inches) (a) Re = 128,000 (b) Re = 253,000 (c) Re = 350,000 (d) Re = 430,000.

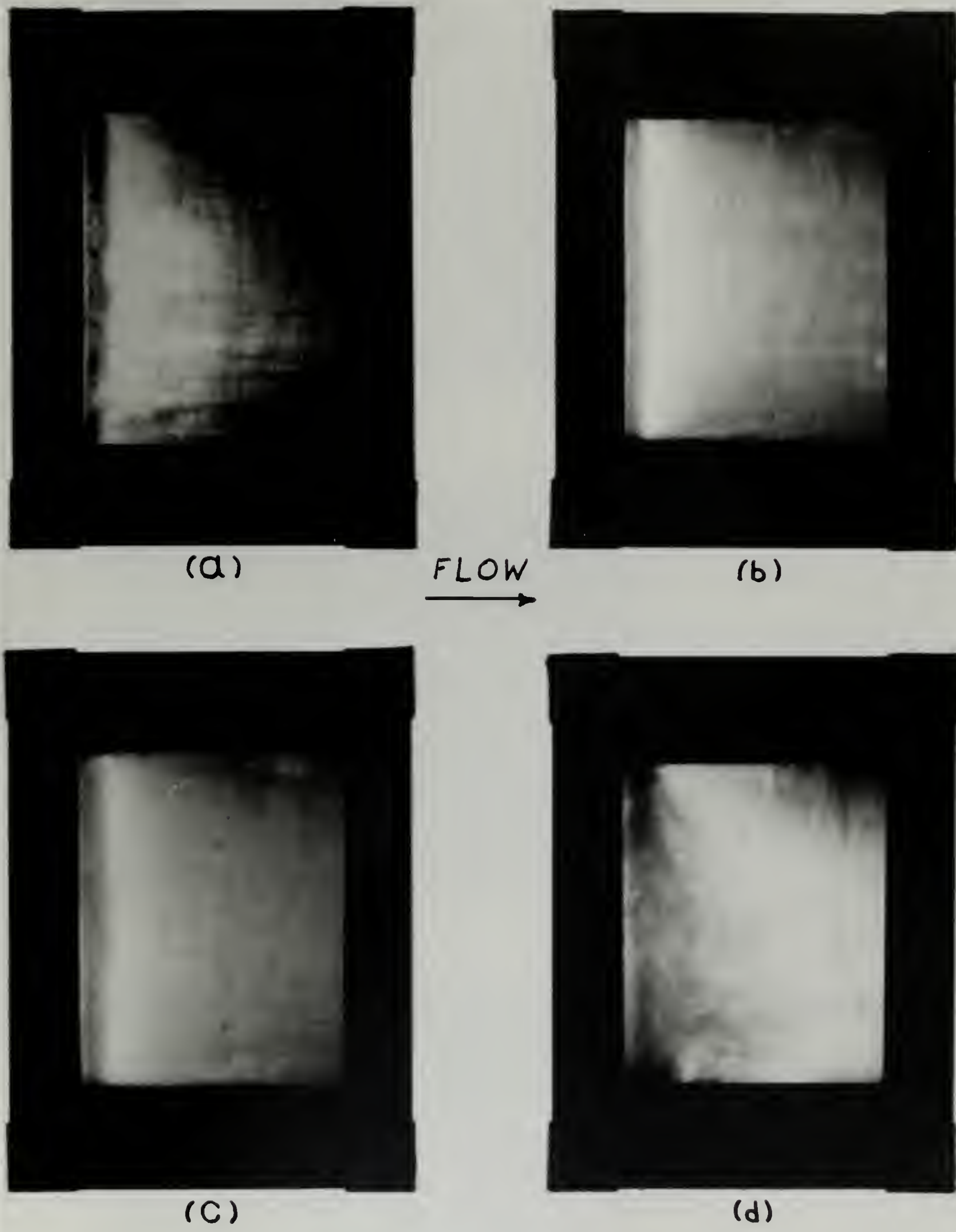
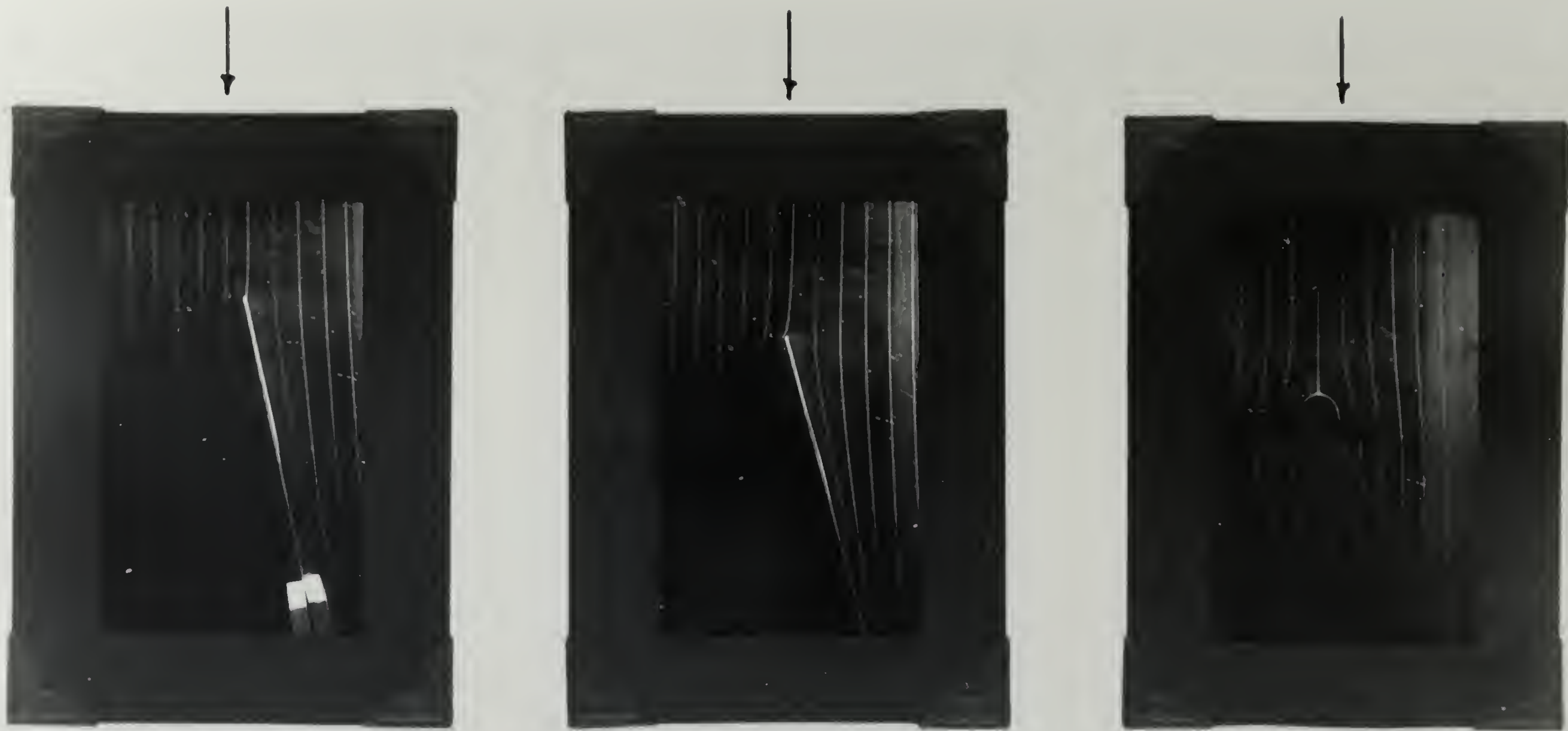


FIGURE 14. Separation on flat plate at various angles of attack. Angle of attack (a) $\alpha = 0.5^\circ$ (b) $\alpha = 2.5^\circ$ (c) $\alpha = 3.2^\circ$ (d) $\alpha = 13.1^\circ$. Reynolds number (based on flat plate length of 5 inches) Re = 128,000 .



(a) (b) (c)

FIGURE 15. Separation on flat plate and cylinder in wind tunnel. Flat plate angle of attack (a) $\alpha = 10^\circ$ (b) $\alpha = 15^\circ$. Reynolds number (based on flat plate length of 5 inches and cylinder diameter of 1.5 inches) (a) Re = 2,400 (b) Re = 2,400 (c) Re = 735.

D. Transition

In this investigation an attempt was also made to visually observe transition from laminar to turbulent flow by applying the technique of electrochemiluminescence. Using this technique both Howland, et al.,[2] and Schiller[3] observed glow patterns that could be interpreted as transition. Howland, et al.,[2] conducted tests of flow over a hydrofoil at various angles of attack. Some of their observed flow patterns resembled the transition spots obtained by Emmons,[8] and some of their patterns were identified as transition streamers. Schiller[3] studied flow patterns about turbulence stimulators similar to those used on ship hull models. He assumed turbulence existed and that his picture was typical of what the technique would show. The present objective was to determine whether or not the process of electrochemiluminescence could provide information directly related to transition.

It was decided to investigate classical transition on a flat plate at zero angle of incidence. Hansen[10] experimentally determined the critical Reynolds number at which transition could first occur. His critical Reynolds number (based on distance to the onset of transition) is 3.2×10^5. According to Schlichting[7] this number should be considered the minimum required for transition. The experiments of Schubauer and Skramstad[11] indicate that at low turbulence, which probably exists in this apparatus, the onset of transition occurs at a $Re = 2.8 \times 10^6$ (based on distance from leading edge to onset of transition) and that fully turbulent flow should exist at $Re = 3.8 \times 10^6$.

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In view of Hansen's[10] critical Reynolds number, it was presumed that transition could be obtained on a long flat plate. Therefore, a

1/4-inch plexiglass plate was constructed 2 inches wide and 30 inches long. The anode was made up of six platinum foil sections each 1 inch wide and 5 inches long. These sections were glued end to end with 1/4-inch overlap. This construction provided an anode 28.5 inches long. The cathode of 1/16-inch aluminum plate was 2 inches wide and 30 inches long. It was glued to the back of the plexiglass plate. The test piece was placed in the 4-inch test section at zero angle of incidence. Since the maximum available velocity in this test piece is 10.8 feet per second, the maximum obtainable Reynolds number is 2.47×10^6 (based on 28.5-inch length).

Observations were made throughout the range of available Reynolds numbers of 4×10^3 to 2.47×10^6. Nothing unusual was seen that could be directly related to transition. However, three phenomena were noted. First, the glow intensity did increase as the flow velocity increased. Second, around a few rough spots the intensity was greater than elsewhere on the plate. The contrast of glow diminished as the velocity increased. These rough spots existed near where the foil sections overlapped. The spots resulted from difficulties experienced with gluing the platinum foil smoothly to the plate. Turbulent flow probably existed near these rough areas. If this is true then the above observation agrees with Schiller's[3] observation of flow about turbulence stimulators. Therefore, it can be concluded that for some range of Reynolds numbers (with the upper limit undetermined), a short region of turbulent flow is indicated by increased glow intensity. The third phenomenon concerns streamers or lines which appeared on the long flat plate. Similar lines were observed on Schiller's[3] flat plate and on the

1/4-inch plexiglass plate was constructed 2 inches wide and 30 inches long. The mesh was made up of six platinum foil sections each 1 inch wide and 5 inches long. These sections were glued end to end with 1/4-inch overlap. This construction provided an anode 28.5 inches long. The cathode of 1/16-inch aluminum plate was 2 inches wide and 30 inches long. It was glued to the back of the plexiglass plate. The test plate was placed in the 6-inch test section at zero angle of incidence. Since the maximum available velocity in this test plane is 10.5 feet per second, the maximum obtainable Reynolds number is 2.47×10^6 (based on 28.5-inch length).

Observations were made throughout the range of available Reynolds numbers of 4×10^3 to 2.47×10^6. Nothing unusual was seen that could be directly related to transition. However, three phenomena were noted. First, the glow intensity did increase as the flow velocity increased. Second, certain rough spots the intensity was greater than elsewhere on the plate. The contrast of glow diminished as the velocity increased. These rough spots existed near where the foil sections overlapped. The spots resulted from difficulties experienced with gluing the platinum foil smoothly to the plate. Turbulent flow probably existed near these rough areas. If this is the case the above observation agrees with Schlichting's[7] description of flow about surface roughness elements. Therefore, it can be concluded that for some range of Reynolds numbers (with the upper limit undetermined), a short region of turbulent flow is indicated by increased glow intensity. The third phenomenon consisted of a network of lines which appeared on the long flat plate. Similar lines were observed on Schlichting's[3] flat plate and on the short plate used in the separation studies. At low Reynolds numbers the lines appeared to originate at the leading edge. For higher Reynolds numbers beginning at about Re = 1×10^6, they appeared to originate in the reattachment region. These lines were continuous and demonstrated a slight horizontal wavy motion. It is probable that the streamers observed by Howland, et al.,[2] were similar to these lines. Since these lines appeared even at subcritical Reynolds numbers, it is unlikely that Howland, et al., observed transition streamers.

One possible reason why transition was not observed in the first test is that the Reynolds number was too low. It was decided to exceed the Schubauer and Skramstad[11] critical Reynolds number of 2.8×10^6 by increasing the flow velocity. This was accomplished by reducing the test section diameter to 2 inches. Even though the contraction nozzle was not designed for this small section, a maximum velocity of 27.5 feet per second is obtained. This corresponds to Re = 6.3×10^6 (based on length of platinum), which even exceeds the Schubauer and Skramstad[11] Reynolds number for fully turbulent flow of 3.8×10^6.

For the second test the width of the flat plate was reduced by 1/8 of an inch to fit inside the 2-inch test section. The dimensions of the platinum section remained the same. The test piece was placed in the test section at zero angle of attack. Observations were made at Reynolds numbers from about 2×10^5 to 6.3×10^6. As in the first test no phenomenon was observed that could be related to transition.

Since the tests described above did not provide information concerning transition, one final test was considered. Howland, et al.,[2] had demonstrated the feasibility of making wake studies with the short plate used in the experiment studied. As the Reynolds numbers the lines appeared to originate at the leading edge. For higher Reynolds numbers beginning at about Re = 1×10^5, they appeared to originate in the recirculation region. These lines were continuous and demonstrated a slight horizontal wavy motion. It is probable that the phenomenon observed by Ireland, et al.,[9] were similar to these lines. Since these lines appeared even at relatively low Reynolds numbers, it is unlikely that Ireland, et al., observed transition phenomena.

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For the second test the width of the flat plate was reduced by 1/2 of an inch to fit inside the 6-inch test section. The dimensions of the platform section remained the same. The test piece was placed in the test section at zero angle of attack. Observations were made at Reynolds numbers from about 2×10^5 to 6.3×10^5. As in the first test, no phenomenon was observed that could be referred to transition.

Since the tests described above did not provide information concerning transition, the final test was considered. Ireland, et al.,[9] had demonstrated the feasibility of making tests similar with the electrochemiluminescent technique. The wake studies of Hama, Long, and Hegarty[9] have provided excellent information pertaining to the phenomenon of transition. Therefore, the present objective was to determine the feasibility of making a transition wake study using the technique of electrochemiluminescence.

Hama, Long, and Hegarty[9] investigated flow over a trip wire on a flat plate at zero angle of incidence. The wire outside diameter was 1/16 inch, and the shortest plate used was 23 inches long. Flow patterns were obtained by injecting chrome blue or methylene blue dye in a narrow slit upstream of the trip wire. The test model was towed in a tank 2 feet wide, 2 feet deep, and 12 feet long. The towing velocities did not exceed 1 foot per second, and the Reynolds numbers varied from 260 to 560. With this technique Hama, et al., were able to observe the development of vortices behind a trip wire for a length of about 10 cm.

In order to investigate flow over a trip wire in the Schiller[3] apparatus, a test piece was constructed consisting of a stainless steel rod as the anode and an aluminum plate as the cathode. Stainless steel was considered for the anode material because it was more available and cheaper than platinum and because it had been observed to glow in previous tests. The rod outside diameter is 1/16 inch. It is insulated from the plate by electrical tape, and it is located 6 inches from the leading edge. The 1/16-inch thick aluminum plate is 2 inches wide and 12 inches long. The test piece was placed in the original 4-inch test section. This was done because this test section provides better control of low velocity flow than the 2-inch

section. The resulting available Reynolds numbers varied from 400 to 5000. The plate was aligned at zero angle of attack.

The results of this test provided little of interest concerning transition. The flow velocity and applied voltage were both varied, but the wake length was never longer than 1/8 of an inch. This short wake was only observed at the low Reynolds number of about 450. This test did prove one thing of significance. The glow on stainless steel was as intense as the glow on platinum foil. Even the line of separation was observed at high Reynolds numbers. The use of stainless steel as the anode material is promising and deserves more investigation.

In these tests the naked eye was the sole detector of light intensity. It was observed that glow intensity increased as the flow velocity increased up to about $Re = 4 \times 10^6$. Above this Reynolds number the glow appeared uniform along the plate and did not appear to the naked eye to change with increased flow velocity. Since Springer[4] has shown that the electrochemiluminescent glow intensity is a direct function of mass transfer and since transition is accompanied by an increase in mass transfer toward the boundary, it may still be possible to detect the region of transition with an instrument such as a photo-multiplier tube.

sections. The resulting available Reynolds numbers varied from 400 to 2000. The plate was aligned at zero angle of attack.

The results of this test provided little of interest concerning transition. The flow velocity and applied voltage were both varied, but the wake length was never longer than 1/8 of an inch. This short wake was only observed at the low Reynolds number of about 400. This test did prove one thing of significance. The glow on stainless steel was as intense as the glow on platinum foil. Thus the glow of separation was observed at high Reynolds numbers. The use of stainless steel as the anode material is promising and deserves more investigation.

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IV. CONCLUSIONS

The evidence presented in this study has shown that lines of separation can be readily observed in laminar flow using the technique of electrochemiluminescence. A study of the applicability of this technique to separation in turbulent flow would be desirable.

Reattachment of fluid flow was observed on the flat plate. The technique of electrochemiluminescence promises to be a useful tool in the study of this phenomenon.

It is concluded from the tests conducted that transition can not be observed by the naked eye using the electrochemiluminescent technique. However, turbulent flow may be indicated by an increase in glow intensity. This phenomenon could be investigated by light intensity measurements with an instrument such as a photomultiplier tube.

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APPENDICES

APPENDIX A

Details of Chemiluminescent Solution

SUBSTANCE	AMOUNT	PURPOSE
H_2O	- - - - -	Solvent
KCl	1.0 Normal	Supporting electrolyte
KOH	0.01 Normal	Adjust pH
H_2O_2	8.5×10^{-4} Normal	Oxidizing agent
Luminol*	4.4×10^{-3} Normal	Chemiluminescent substance

The kinematic viscosity of the solution was assumed to be the same as water at 70 °F.

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*Eastman Kodak Company, Luminol = (5-amino-2, 3-dihydro-1, 4-phthalazinedione)

APPENDIX B

Literature Survey of Flow Separation

I. Background

The boundary layer equations have defied exact solution except for the most simple cases. Phenomena of interest, such as the point of separation in a given flow, cannot be computed exactly. There are approximate solutions to the boundary layer equations, and the use of these has led to methods of predicting the point of separation for fluid flow over a body with reasonable engineering accuracy.

For the case of two-dimensional laminar external flow over a body, approximate solutions for the separation point have been worked out by Thwaites[12] and Stratford.[13] Thwaites' method consists of linearizing the Navier-Stokes equation and examining experimental data to determine appropriate equation constants. Thwaites empirically determined a value of his equation parameter m, for which separation would occur ($m = 0.082$). The parameter $m = -\frac{\theta^2 dU}{\nu\, dx}$, where θ is the momentum thickness, $U(x)$ is the velocity in the free stream outside of the boundary layer, x is a coordinate taken along the surface of the body, and ν is the kinematic viscosity of the fluid. In this method it is assumed that $U(x)$ is known.

Stratford[13] divides the boundary layer into two parts: an inner layer where viscosity is important and an outer layer where viscosity is unimportant and the flow is almost ideal. Stratford's method requires the solution of a single equation for x_s, the point along the surface of the body where separation occurs. The assumption used is that $\frac{\partial^2 p}{\partial x^2} = 0$,

where x is a coordinate along the surface of the body, and p is the static pressure. The join between the inner and outer layers is postulated to be continuous in u, $\partial u/\partial y$, $\partial^2 u/\partial y^2$, and ψ, the stream function. Here u is the velocity inside the boundary layer in the x direction, and y is a coordinate perpendicular to the surface of the body. The solution for the outer layer is obtained by superimposing solutions of Bernoulli's equation for ideal flow and the Blasius solution of the boundary layer with no pressure gradient. In the inner layer where the viscous forces are very large, a solution is obtained by equating the viscous and pressure forces. The solutions for the inner and outer layers and the postulated boundary conditions are enough to determine the velocity profile and the equation for separation except for a coefficient which is dependent upon $\partial^2 p/\partial x^2$. The value of the coefficient can be changed as appropriate by empirical methods for $\partial^2 p/\partial x^2$ other than zero. Stratford's method requires a knowledge of $U(x)$ and the pressure distribution along the body.

Curle and Skan[14] have modified Thwaites' method by changing the value of the parameter determining separation to m = 0.090. The Curle and Skan modification to the Stratford method consists of changing the empirical coefficient in the Stratford equation. Curle and Skan indicate that the Stratford method is slightly the more accurate of the two, but it cannot be used to compute such boundary layer parameters as the momentum thickness (θ), which can be calculated using Thwaites' method.

Since it will be of interest to know θ at the point of transition to turbulence and since the accuracies of Thwaites' and Stratford's

methods are not very different, it was decided to use Thwaites' method as modified by Curle and Skan as the primary means of predicting the laminar flow separation point. Thwaites' method is described in more detail in the following section.

For the case of turbulent external flow over a body, numerous methods (e.g., Refs. 15-20) have been developed for predicting the separation point. Among these are the work of Von Doenhoff and Tetervin[16] in determining general equations for boundary layer parameters such as θ and predicting the point of separation of fluid flow over a body. The criterion they used for predicting separation was that separation occurs when the shape parameter $H = 2.6$ where $H = \delta^*/\theta$ and δ^* is the displacement thickness. This criterion was confirmed experimentally by Schubauer and Klebanoff.[21]

Townsend[17] and Stratford[18] have developed methods of calculating the separation point in turbulent flow. Their methods are based on the pressure distribution along a body and its behavior at or near separation.

Maskell[19] developed a method for calculating the turbulent boundary layer for which the separation criterion is the requirement that the skin friction (C_f) goes to zero at the point of separation. This method utilizes the Ludwieg-Tillmann[22] skin friction formulation considered by others (Refs. 15, 18, and 23) as probably the best available. Sanborn[15] concurs in the accuracy and usefulness of Maskell's method.

It was decided to concentrate primarily on laminar flow in this study, but if time and the experimental apparatus permitted, turbulent flow would be treated also. For turbulent flow Maskell's method appears

to be most accurate while Stratford's method is simpler to apply. Stratford's method should then be a good check on the results as given by Maskell.

II. Brief Description of the Method of Thwaites[12] for Predicting the Point of Separation in a Two-Dimensional Flow Over a Body

The Navier-Stokes equation with Prandtl's approximations for a two-dimensional boundary layer may be written as:

$$\frac{d\theta}{dx} = -(H + 2)\,\frac{dU}{dx}\,\frac{\theta}{U} + \frac{\nu}{U^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{1}$$

The momentum thickness (θ) is given empirically (Ref. 12) by:

$$\theta^2 = 0.45\,U^{-6}\,\nu \int_0^x U^5\,dx \tag{2}$$

Define the parameters m, n, and L as follows:

$$m = -\frac{\theta^2}{\nu}\,\frac{dU}{dx} \tag{3}$$

$$n = \frac{\theta}{U}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{4}$$

$$L = \frac{U}{\nu}\,\frac{d(\theta^2)}{dx} \tag{5}$$

Taking $\frac{d(\theta^2)}{dx} = 2\,\theta\,\frac{d\theta}{dx}$ and rearranging (1) gives:

$$\frac{d(\theta^2)}{dx} = 2(H + 2)\,\frac{m\nu}{U} + \frac{2\nu}{U}\,n \tag{6}$$

Rearranging (6) gives:

$$L = 2\left[m(H + 2) + n\right] \tag{7}$$

Thwaites found equation (7) to be nearly linear in m and chose $L = 0.45 + 6m$. By inspecting experimental data he determined that separation should occur at $m = 0.082$. The Curle and Skan[14] modification gives $m = 0.090$ at separation. Solving equation (7) for H at separation (using $m = 0.090$), $n = 0$ (since $[\partial u/\partial y]_{y=0}$) and $H = 3.5$ which is a more familiar criterion for separation than is the value of the parameter m.

Finally at separation:

$$0.090 = -\frac{\theta^2}{\nu}\frac{dU}{dx} = -\frac{0.45}{U^6}\frac{dU}{dx}\int_0^{x_{sep.}} U^5\,dx \qquad (8)$$

Solving equation (8) for $x_{sep.}$ gives the position of separation in the laminar flow.

Symbols

x, y	Coordinate axes parallel to the surface and perpendicular to it, respectively.
L, m, n	Parameters defined in Section II.
H	Shape parameter; given as δ^*/θ.
θ	Momentum thickness; given as $\int_0^\infty \frac{u}{U}(1 - \frac{u}{U})dy$.
δ^*	Displacement thickness; given as $\int_0^\infty (1 - \frac{u}{U})dy$.
u	Velocity inside the boundary layer in the x direction.
U	Free stream velocity outside the boundary layer.
ν	Kinematic viscosity of the fluid.